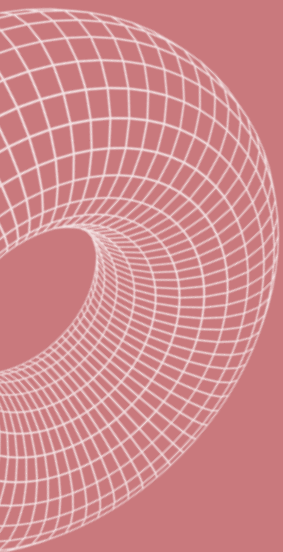


ISSUE FIVE
MAY 2026

CMIT'S OWN

DONUT

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EDITORIAL NOTE

Dear Math Enthusiasts,

Get ready to indulge in a fresh dose of mathematical goodness! We're thrilled to present the fifth issue of Donut, your go-to newsletter for all things math at IISER Thiruvananthapuram. This issue is packed with exciting event details, thought-provoking articles, etc.

Dive into our coverage of recent events, including Pi Week celebrations, Anvesha aficionados, and S6 CMIT's anniversary program. This issue features insightful articles that blend mathematics with physics, cryptography, and more, alongside explorations of computational limits and geometric insights from talented students of IISER Kolkata and our institution. Plus, don't miss our archive of talks from the recent Talk Series, featuring brilliant minds sharing their passion for mathematics.

We're passionate about creating a community that celebrates the beauty and wonder of mathematics. Your feedback is the sprinkles on the donut. Please share your thoughts, opinions, and suggestions with us, and help shape the future of Donut. You can submit anonymous feedback here:

[This is a hyperlink](#)

Warm regards,
The Content Creation Team of CMIT



SIXTH FOUNDATION ANNIVERSARY

Celebrating Six Years of CMIT!

As CMIT marks its sixth anniversary, we are thrilled to present S6, a specially curated talk series that brings together eminent mathematicians to share their insights and stories. This celebratory edition is more than just a lecture series—it's a journey through the rich and evolving landscape of mathematics. From classical geometry to modern algebraic structures, and from ancient Indian insights to elegant combinatorial patterns, S6 invites both enthusiasts and experts to engage with mathematics in new and thought-provoking ways.

Dr. Vaibhav Vaish from IISER Mohali opens the series with a deep dive into the historical trajectory of Pappus' Theorem, tracing its influence from Euclidean and Cartesian geometry to its surprising modern relevance in algebraic geometry. Following this, Dr. Manjil Saikia of Ahmedabad University explores the enchanting world of Catalan numbers, revealing the surprising web of bijections and structures that this humble sequence connects in combinatorics. Lastly, Dr. Shailesh Shirali takes us on a multifaceted tour across number theory, analysis, and ancient Indian mathematics, featuring topics that span elegant iterations, geometric enumerations, and unsolved historical puzzles.



ANVESHA AFICIONADOS

Cyclotomic Polynomials and the Roots of Unity

Sankeerth Kathiran (B24), Rithik Menon (B24)

Roots of Unity

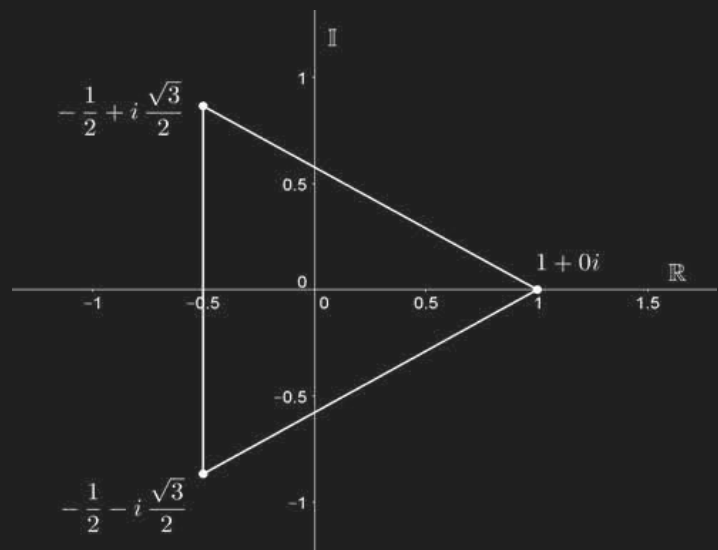
Have you ever wondered what happens when you raise a complex number to a power and it turns into 1? That's exactly what the roots of unity are all about. These special numbers come up when we solve equations like $x^n = 1$, and they have some pretty cool patterns, especially when you look at them on the complex plane.

Of the roots of unity, the most studied roots are the cube roots of unity, which we are all familiar with and have learned in high school. These are the solutions to the equation $x^3 - 1 = 0$, and they are often used in examples and exercises to illustrate the properties of complex numbers and polynomials.

The cube roots of unity are 1 , ω and ω^2 , where $\omega = \frac{-1+i\sqrt{3}}{2}$. (Note: i is the imaginary unit, defined as $i = \sqrt{-1}$.)

$\omega = \frac{-1+i\sqrt{3}}{2}$ gives ω^2 to be equal to $\frac{-1-i\sqrt{3}}{2}$.

Cyclotomic polynomials are closely related to the roots of unity. The n^{th} cyclotomic polynomial is the polynomial whose roots are the n^{th} primitive roots of unity.



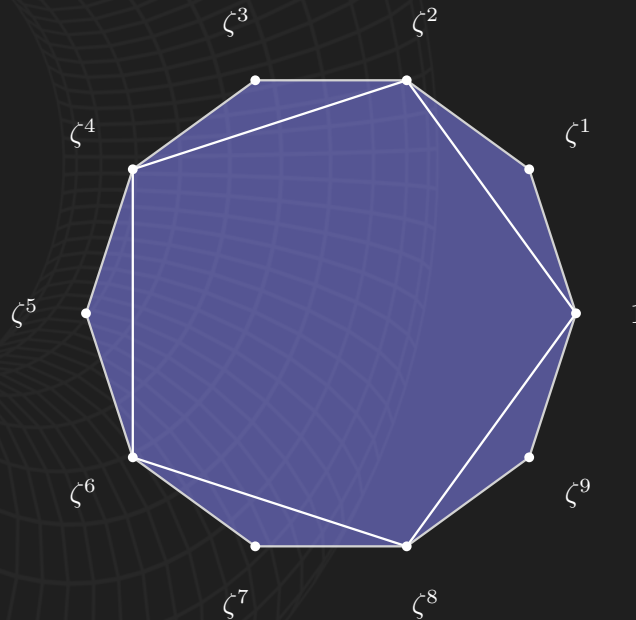
Primitive Roots of Unity

For an integer n , the primitive n^{th} roots of unity are the n^{th} roots of unity that are not the k^{th} roots of unity for any $k < n$.



Example: The roots of $x^4 - 1 = 0$ are $1, -1, i$ and $-i$. Among these, the primitive 4th roots are only i and $-i$.

Audience Interaction



A visual depiction of the 10th roots of unity, and the polygon formed (in this case, a pentagon) by connecting the 10th primitive roots of unity on the complex plane.

Question: Why are 1 and -1 not primitive 4th roots of unity?

Answer: 1 is the 1st primitive root of unity since it is the root of the equation $x - 1 = 0$.

-1 is the 2nd primitive root (as $x^2 - 1 = 0$ gives $x = 1$ and $x = -1$ as solutions, and $x = 1$ satisfies $x - 1 = 0$). In other words, 1 and -1 satisfy $x^k - 1 = 0$ for some $k < 4$, hence they are not considered to be primitive 4th roots of unity.

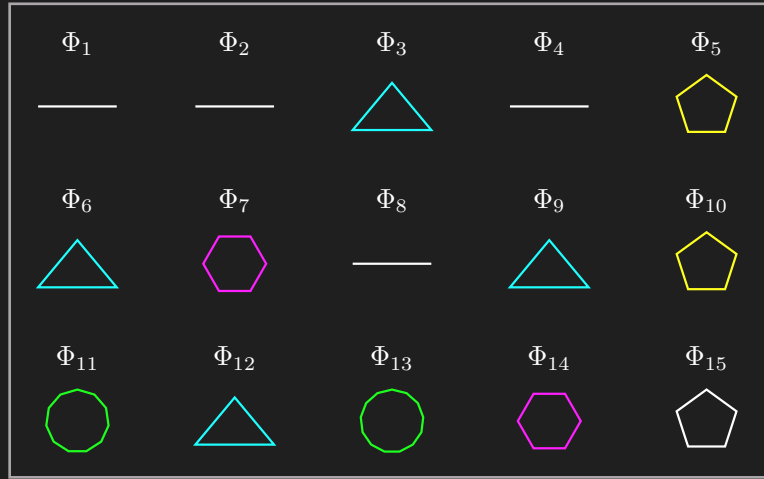
Essentially, the primitive n^{th} roots of unity are the complex numbers,

$$e^{i\frac{2\pi k}{n}}; 1 \leq k < n \text{ and } \gcd(k, n) = 1.$$

Returning to our topic, which is cyclotomic polynomials, the n^{th} roots of unity lie on the unit circle as the vertices of a regular n -gon polygon. If,

$$\zeta_n = e^{\frac{2i}{n}} = \cos\left(\frac{2\pi}{n}\right) + i \cdot \sin\left(\frac{2\pi}{n}\right)$$

then the roots consist of $1, \zeta_n, (\zeta_n)^2 \dots (\zeta_n)^{n-1}$.



For cyclotomic polynomials upto (not including) $\Phi(15)$, their roots when connected form regular polygons on the complex plane (Note that in the case of $\Phi(15)$, the polygon formed is an irregular heptagon).

A fundamental relation involving cyclotomic polynomials is,

$$\begin{aligned} x^n - 1 &= \prod_{1 \leq k \leq n} \left(x - e^{2i\pi \frac{k}{n}} \right) \\ &= \prod_{d|n} \prod_{1 \leq k \leq n, \gcd(k,n)=d} \left(x - e^{2i\pi \frac{k}{n}} \right) \\ &= \prod_{d|n} \Phi_{\frac{n}{d}}(x) = \prod_{d|n} \Phi_d(x). \end{aligned}$$

One can also express the n th cyclotomic polynomial in terms of the Mobius function $\mu(n)$,

$$\Phi_n(x) = \prod_{d|n} (x^d - 1)^{\mu(\frac{n}{d})} = \prod_{d|n} (x^{\frac{n}{d}} - 1)^{\mu(d)},$$

Where,

$\mu(n) = 1$, if $n = 1$ or n has an even number of prime factors,

$\mu(n) = -1$ if n has an odd number of prime factors, and

$\mu(n) = 0$ if n is a multiple of a square of a prime number.

For example,

$$\begin{aligned} (8) &= (x^8 - 1)^{\mu(1)} * (x^4 - 1)^{\mu(2)} * (x^2 - 1)^{\mu(4)} * (x - 1)^{\mu(8)} \\ &= (x^8 - 1)^1 * (x^4 - 1)^{-1} * (x^2 - 1)^0 * (x - 1)^0 = \frac{[x^8 - 1]}{[x^4 - 1]} = (x^4 + 1) \end{aligned}$$

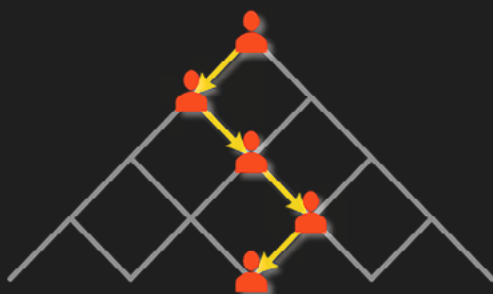
Cyclotomic polynomials give us a powerful way to understand the structure of the roots of unity, especially the primitive ones. Even though they come from complex numbers, these polynomials always have neat, integer coefficients and show up in many areas of mathematics like number theory, algebra, and even coding theory. By breaking down $x^n - 1$ into simpler parts, cyclotomic polynomials help us see patterns and relationships that aren't obvious at first glance. They're a great example of how deep ideas in math can come from something as simple as solving $x^n - 1 = 0$.

QUANTUM WALK

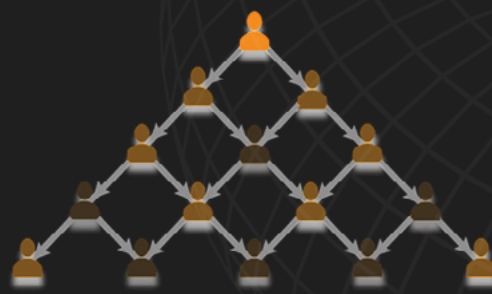
Prerana Chatterjee (B23), Pranav Gupta V (B23),
Neona Maria Shaji (B23), Pulckit Anand (B23),
Arnobick Polley (B24), Urvashi Srivastava (B24)



Quantum computation holds the promise of revolutionizing the way we process information and one of its intriguing manifestations is the quantum walk — the quantum analog of the classical random walk. In this project, we explored the potential of quantum walks to outperform classical walks for computational tasks. Our goal was to demonstrate how quantum walks when implemented with quantum logic gates and algorithms like Grover's, lead to significant speedups compared to their classical counterparts. The project was implemented using IBM's Qiskit framework, which provided a versatile platform to simulate and test our quantum circuits.



(a) Random Walk



(b) Quantum Walk

Classical Operation and Random Walks

A classical random walk is a process where a particle moves step-by-step in a space (often represented as a line or graph), with the direction of each step determined probabilistically. This process has applications

in search algorithms, diffusion models, and network theory. However, classical random walks spread over the search space slowly: the standard deviation of the position after n steps scales as $\sqrt{n}$. This limits their efficiency in exploring large datasets or decision trees.

Quantum Walks

A quantum walk introduces quantum superposition and interference to the walk process. Here, the “walker” exists in a superposition of positions, and the movement is governed by unitary operations rather than probabilistic rules. As a result, the quantum walk spreads quadratically faster than the classical one, with the standard deviation scaling as n . This property opens the door to faster computation, especially in search and graph traversal problems.

Quantum Logic Gates and Circuit Design

In our project, we constructed quantum walk circuits using fundamental quantum logic gates:

- **Hadamard (H) gate:** Creates superpositions.
- **Pauli-X (NOT) gate:** Flips qubit states.
- **Controlled-NOT (CNOT):** Essential for entangling qubits.
- **Rotation gates (Ry, Rz):** Used to define custom coin operators in walks.
- **Swap and Toffoli gates:** Useful in multi-qubit control and walk evolution.

The coin operator and shift operator — key components of the quantum walk — were implemented using combinations of these gates. The coin operation determines the walker’s direction, while the shift operation moves the walker accordingly.

Grover’s Algorithm

To demonstrate practical speedup, we integrated the quantum walk with Grover’s Search Algorithm, which offers a quadratic speedup over classical search algorithms. Grover’s algorithm operates on the principle of amplitude amplification and can be interpreted as a specific form of quantum walk over an unstructured database.

In our implementation:

- The quantum walk initialized the system in a superposition.
- Oracle and diffusion operators were applied to mark and amplify the

target state.

- The number of iterations followed the Grover bound: $\sim\sqrt{N}$, where N is the size of the search space.

This allowed us to simulate a search problem (e.g., finding a marked item in a list), showing how the quantum walk facilitated faster convergence to the solution compared to a classical approach.

Logic Speedup: Classical vs Quantum

We compared the time complexity of a classical walk-based search and the quantum walk implementation using Qiskit simulations. The classical walk required $O(n)$ steps to find a target item, while the quantum walk with Grover-inspired logic found the solution in approximately $O(\sqrt{n})$ steps, consistent with theoretical predictions.

This *logic speedup* was evident in:

- Circuit depth and gate count analysis
- Probability distribution after measurement
- Simulation time on classical hardware emulating quantum systems

Even with noise and limited qubits on IBM's quantum backends, the quantum walk demonstrated noticeable efficiency in reaching target states.

Implementation with Qiskit

IBM's Qiskit framework was instrumental in realizing this project. Key components of our implementation included:

- Qiskit Terra: For circuit construction and gate-level control.
- Qiskit Aer: To simulate quantum circuits and analyze outcomes.
- Qiskit Visualization: To plot state vectors and probability histograms.

A custom function was developed to perform multiple-step quantum walks, incorporating dynamic coin operators and flexible walk topologies (e.g., line, cycle). The final circuits were modular, allowing us to scale the walk to higher dimensions or connect it to more complex oracles.

Conclusion

This project demonstrated the computational advantage of quantum walks over classical random walks in solving search problems. By

integrating quantum logic gates, Grover's algorithm, and the Qiskit toolkit, we built and tested circuits that showed real quantum speedup in theory and simulation. While current quantum hardware has limitations, our results highlight the promise of quantum walks in future algorithms — especially for database searching, optimization, and quantum simulation tasks.

As quantum processors evolve, quantum walks may become a core component of quantum computing's practical toolkit, enabling solutions to problems that are intractable classically.

Study of Growth Models in Nature

Pranav M (B23) and Aneeth Kumaar P V (B23)

The project "Study of Growth Models in Nature" explores the mathematical foundations and real-world applications of natural growth and decay processes, with a central focus on the mathematical constant Euler's number 'e'. It delves into how exponential models, often governed by 'e', explain a wide range of physical, biological, and chemical phenomena.

Bernoulli's Compound Interest Problem

The study begins with Jacob Bernoulli's exploration of compound interest, one of the earliest mathematical formulations of growth. Bernoulli investigated how a principal amount grows over time when interest is compounded more and more frequently. He discovered that as the number of compounding intervals increases indefinitely, the final amount approaches a limiting value. This limit is the mathematical constant $e \approx 2.71828$, which arises naturally when interest is compounded continuously.

Invention of Euler's Number 'e'

The concept of 'e' was further formalized by Leonhard Euler, who recognized its foundational role in the calculus of exponential growth. Euler's work unified earlier ideas and gave rise to the exponential function $f(x) = e^x$, which possesses the unique property that its derivative is equal to itself. This made 'e' indispensable in the modeling of growth and decay across scientific disciplines. The invention of 'e' was a pivotal moment in the history of mathematics, influencing

differential equations, probability, and financial mathematics.

'e' in Nature

Euler's number is not just a mathematical curiosity, it appears pervasively in natural systems. From the distribution of particles, bacterial colony growth, and radioactive decay, to more abstract systems such as information entropy and chaotic systems, 'e' serves as a fundamental descriptor of how systems evolve over time. The project emphasizes how natural growth processes often follow exponential patterns.

'e' in Growth and Decay Models

The core of the project is an in-depth exploration of how 'e' governs both growth and decay models in various scientific fields:

Damped Simple Harmonic Motion (SHM)

In systems experiencing resistance (like a pendulum in the air or a spring-mass system with friction), the amplitude of oscillation decreases over time. This decay follows an exponential law:

$$A(t) = A_0 e^{bt}$$

where b is the damping constant. The use of 'e' in this context models energy dissipation in mechanical systems.

Freundlich Isotherms and Rate Equations of Reactions

In chemistry, 'e' appears in rate equations describing how fast reactions proceed. For first-order reactions, the concentration of a reactant decays exponentially:

$$[C](t) = [C]_0 e^{kt}$$

where k is the rate constant. Similarly, Freundlich isotherms, which describes adsorption on surfaces, incorporates exponential relationships that emerge from energy interactions at molecular levels.

Population Growth Models

In ecology and demography, the exponential growth model describes how a population grows under ideal conditions,



$$P = P_0 e^{rt}$$

where r is the growth rate. This model also explains epidemic spread, financial inflation, and resource depletion when unchecked by limiting factors. It can be extended into the logistic model, which introduces carrying capacity and still retains 'e' in its formulation.

Conclusion

The project concludes that Euler's number 'e' is a universal constant underlying the structure of growth and decay in both natural and engineered systems. From the compounding of wealth to the oscillations of mechanical systems and the growth of living populations, 'e' serves as a bridge between mathematics and the natural world. Understanding its role enables deeper insights into dynamic processes across disciplines such as physics, chemistry, biology, and economics. The study highlights how abstract mathematical concepts, like 'e', become powerful tools for modeling real-world complexity.

Applications of graph theory in Landscape ecology

M Rishika (B23)

Applying Graph Theory to Landscape Ecology:

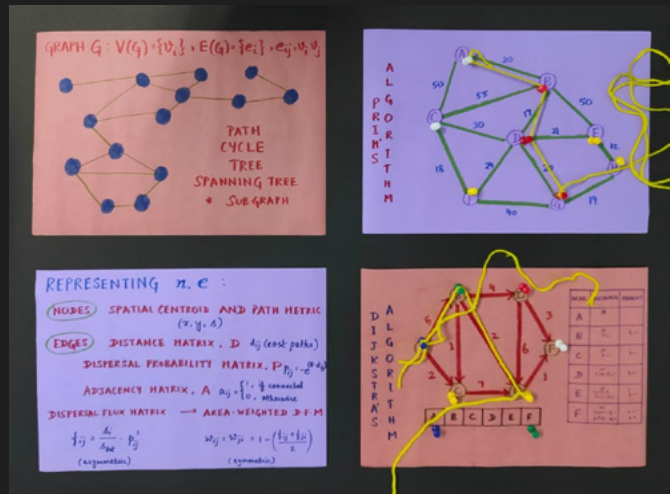
I explored the applications of graph theory in landscape ecology, revealing a powerful tool for understanding and conserving complex ecosystems. By representing habitats as nodes and possible traversals between them as edges, landscape graphs can be constructed to analyze the structure and function of ecosystems.

Modeling Landscape Graphs

In this framework, nodes are characterized using spatial centroids and path metrics, while edges are represented using various matrices, such as adjacency, distance, dispersal probability, or area-weighted dispersal flux matrices. This approach enables researchers to quantify the relationships between habitat patches and identify key connectivity pathways.

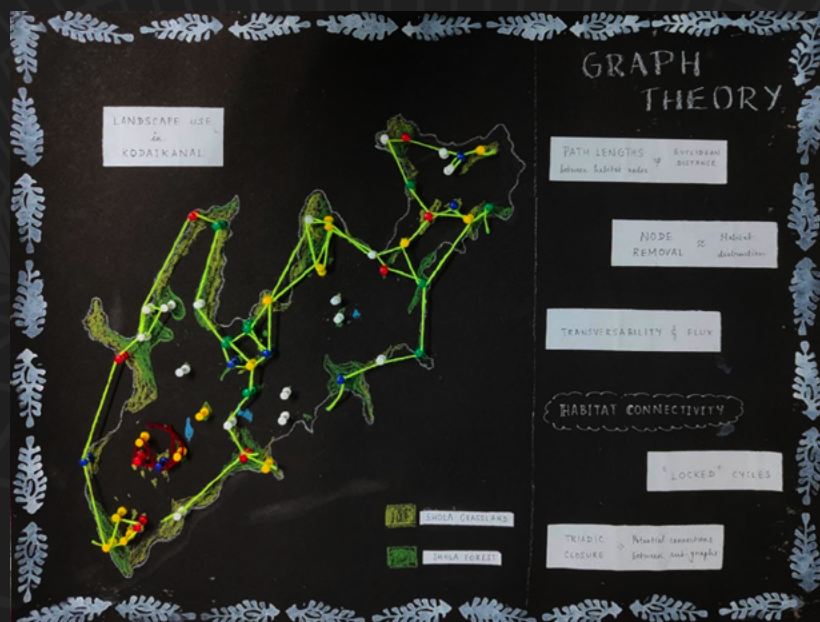
Hands-on Demonstrations

To illustrate the practical applications of graph theory in landscape ecology, hands-on demonstrations of two fundamental algorithms were conducted: Prim's algorithm for finding the minimum spanning tree and Dijkstra's algorithm for determining the shortest path between two nodes. These algorithms have potential implications for conservation planning, particularly in assigning priority to degraded patches in disturbed landscapes.



A Case Study: Kodaikanal's Shola Forests and Grasslands

A theoretical case study was conducted using the land use map of Kodaikanal, a high-elevation human-disturbed landscape in the southern Western Ghats. This region's unique shola forests and grasslands provide a fascinating example of the challenges and opportunities in landscape ecology. By applying graph theory to this system, we can gain valuable insights into the dynamics of ecosystem connectivity and inform effective conservation strategies.

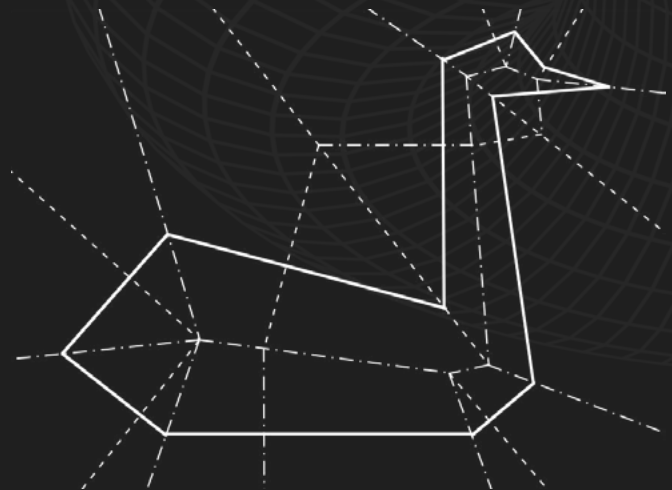


The fold and cut theorem

Christopher (B23), Aaryaman Rastogi (B24), Nilanjan Goswami (B24)

The fold-and-cut theorem states that any straight-edged shape can be made by folding a sheet of paper and making a single straight cut. This idea first appeared in 1721, in the Japanese work *Wakoku Chiyekurabe* by Kan Chu Sen. A generalized version of this theorem states that any set of line segments can be achieved through precise flat-folding, making the theorem both a historical curiosity and a contemporary mathematical marvel.

The proof for the Fold and Cut Theorem can be broken into three major parts. The first step is creating the crease pattern. For simple, symmetric shapes, angle bisectors are usually enough to align the edges naturally. However, asymmetrical shapes require a more nuanced approach, combining both angle and perpendicular bisectors to ensure accurate mirroring of the edges. To generalize this for more complex shapes, one can use a concept called the straight skeleton. To visualize this, imagine shrinking the shape inward, with each edge moving parallel to itself. The trajectory of the vertices as they shrink gives you a straight skeleton. Once the skeleton is formed, skeleton perpendiculars are added to ensure every fold is in place before the final cut.



Straight Skeleton Pattern of a Swan
(Erik Demaine, Professor, MIT (<https://erikdemaine.org>))

The next stage involves deciding whether the creases should be a mountain fold (folding the paper “downwards”) or a valley fold (folding the paper “upwards”). Concepts from differential geometry help provide proof of the following: if a skeleton edge bisects an angle less than 180° , it should be a mountain fold; if it bisects an angle greater than 180° , it should be a valley fold. For edges that bisect exactly 180° , Maekawa’s Theorem comes into play, stating that at any flat-foldable vertex, the difference between mountain and valley folds must be exactly two.

$$|M - V| = 2$$

Finally, to verify that the folded shape will lie flat, Kawasaki’s Theorem may be used. It states that at a vertex with an even number of creases,

the alternating sum of the angles between them must equal zero.

$$\theta_1 - \theta_2 + \theta_3 - \theta_4 + \dots = 0$$



The Crease Pattern of a Scalene Triangle

For example, in an equilateral triangle, the angle bisectors meet at 60° intervals ($\theta_i = 60^\circ$, $1 \leq i \leq 6$), satisfying this condition. However, Kawasaki's Theorem only ensures local flatness, and ensuring the entire folded shape lies flat requires more advanced mathematical concepts, such as Euler's decompositions.



The exhibit aimed to recreate John Conway's demonstration of Rational Tangles

S V Roshin (B22)

It starts with two ropes (the orange lines in the figure) and 4 volunteers (a, b, c , and d) arranged as shown:

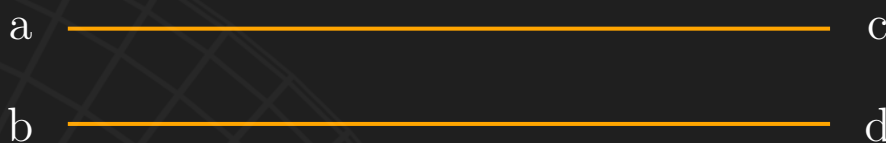


Fig 1. Initial setup

There are 2 permissible actions allowed - Twist (denoted by T) and Rotate (denoted by R).



Fig 2. One twist (T)

Note that there is no “Untwist” or counterclockwise rotation allowed.

The demonstration is as follows: starting from the position in Figure 1, the participants are allowed to use these 2 actions of twisting and rotating as many times as they wish until they are left with a pretty complicated-looking structure. Additionally, all the actions are being noted in the sequence they’ve been carried out. The goal now is to untangle this structure using the very same actions of Ts and Rs.



Fig 3. A Twist and Rotate (TR)

Conway characterizes each particular configuration of these ropes (configurations derived only through T and R operations, of course) using rational numbers. We start from the initial untangled state as zero.

How are these numbers assigned to each state?

Each twist (T) is an addition of 1. This seems pretty intuitive, at least in the beginning, where the number of points of intersection can be counted, and thus a twist adds one to the number associated with the tangle.

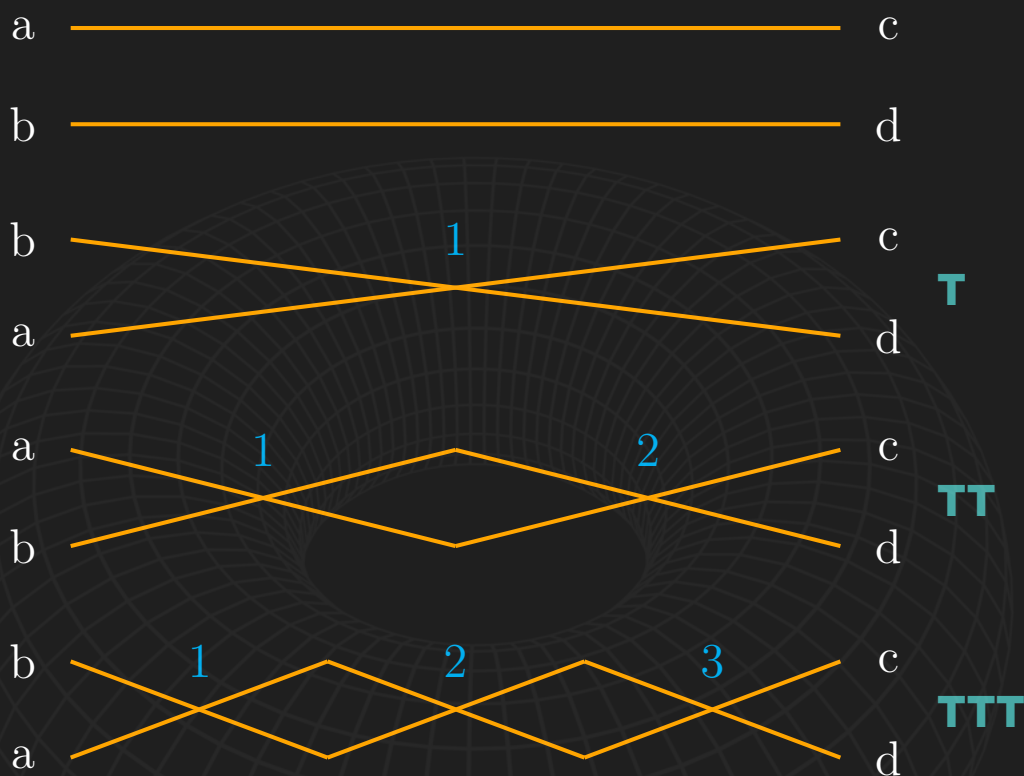


Fig 4. (Action of twists on the tangle number)

What a rotation does to the tangle number is not as straightforward, but can be worked out. A rotation takes the tangle number x and makes it $-1/x$.

Now it is clear how each tangle gets a rational number assigned to it. Untangling it is simply a matter of bringing the tangle number back to zero. This is achieved using Euclid's division lemma.

For example, consider the sequence of actions:

$$TTTRTTR = T^3RT^2R,$$

which gives us the number $(-3/5)$.

$$\begin{aligned} T^3RT^2R &\Rightarrow 3 \xrightarrow{R} -1/3 \xrightarrow{T^2} 5/3 \xrightarrow{R} -3/5 \\ 3 &= 5 \times 1 - 2 & (1) \\ 5 &= 2 \times 3 - 1 & (2) \\ 2 &= 1 \times 2 + 0 & (3) \end{aligned}$$

Here, moving from one equation to the next represents a rotation operation. Rearranging (1),

$$\begin{aligned} -3/5 + 1 &= 2/5 \\ \downarrow \\ \text{rotate } 2/5 &\text{ to get equation (1)} \\ \downarrow \\ -5/2 + 3 &= 1/2 \implies 3 \text{ twists} \\ \downarrow \\ \text{rotate } 1/2 &\text{ to get equation (3)} \\ \downarrow \\ -2 + 2 &= 0 \implies 2 \text{ twists} \end{aligned}$$

To untangle T^3RT^2R we do: TRT^3RT^2

$$-\frac{3}{5} \xrightarrow{T} \frac{2}{5} \xrightarrow{R} -\frac{5}{2} \xrightarrow{T^3} \frac{1}{2} \xrightarrow{R} -2 \xrightarrow{T^2} 0$$

Thus, the given configuration of ropes has been untangled.



It is also a noteworthy observation that the 2 actions Twist and Rotate can be represented as 2x2 matrices.

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

And any rational number p/q can be represented as a 2 x 1 matrix.

$$p/q := \begin{pmatrix} p \\ q \end{pmatrix}$$

Example: $T^2 R = -1/2$, which is basically

$$\begin{aligned} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} &= \begin{pmatrix} -1 \\ 2 \end{pmatrix} \end{aligned}$$

T and R are also generators of $SL_2(\mathbb{Z})$ ($\mathbb{Z}$ - integers). Thus, this entire exercise can be viewed as a group action of $SL_2(\mathbb{Z})$ on $\mathbb{Q}$ (rational numbers), that is, $SL_2(\mathbb{Z}) \times \mathbb{Q} \rightarrow \mathbb{Q}$.

It is pretty interesting to see how group actions can be visualized physically through this simple rope activity, which can be enjoyed regardless of whether one understands the complexities behind it!

Thinking corner!



Consider two real numbers, x and y , randomly chosen from the open interval $(0,1)$. What is the probability that the ratio x/y rounds to an even integer? And what about the probability that x/y approximated down to the nearest integer is even? Might these probabilities be connected to some of the most fundamental constants in mathematics?

Answers:
 $\frac{1}{2}, 1 - \frac{1}{\ln 2}$



Finite Points and Lines at Infinity

Abhratanu Ray (22MS052), IISER Kolkata

1 Introduction

Sometimes, the most fun problems to work on are the ones easiest to visualize. Especially in geometry, problems like this are everywhere. The simplest ideas can generate non-trivial problems, with beautiful solutions.

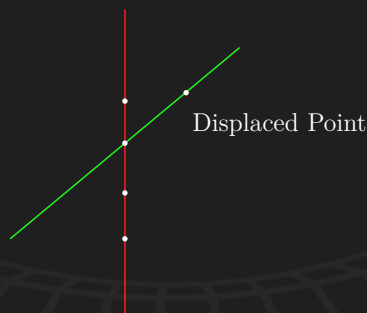
Thinking about random point arrangements (the basis of incidence geometry; that is, the geometry that deals with sets of points and lines through points), one stumbles upon a problem that is easy enough to come up with, which is the following, presented by James Joseph Sylvester in 1893.

Problem

Prove that it is not possible to arrange any finite number of real points so that every straight line through any pair of points shall pass through a third, unless all points lie in the same straight line.

Or rephrased, in every arrangement of a finite number of points on the Euclidean plane, there exists a straight line passing through only two of these points, unless all points are collinear.

It is easy to visualize this. Imagine some points, all in the same line. We know there is some unique line passing through all of them. Now, we displace one of these points slightly.



We can see that some line passing through **only two** of the points has been generated, through the displaced point, and the adjacent point to it, for example. The idea of the problem is to show such a line, passing through **two points**, but not any more, exists in all possible arrangements of finite points.

We shall go through 2 proofs of the problem statement. One given by **Tibor Gallai**, in 1944, after Paul Erdős popularised the problem; and the other much later, in 1986, by **Leroy Milton Kelly**. The former proof is done in quite an interesting way, making use of what is known as **projective geometry**. Both proofs are quite elegant, and in the process of understanding Gallai's proof, we shall have a look into the world of projective transformations and planes.

We list 2 standard definitions for our proofs:

Definition 1

Let $S = \{n_1, n_2, n_3, \dots\}$, where S is of size n ($n \geq 3$) be a set of not all collinear points in the Euclidean plane. A line that contains two or more points of S is called a **connecting line** and is **ordinary**, if it contains exactly two points.

We will talk about the definition of an “affine plane” in Section 3, but for now, we are working in the Euclidean plane, which is an affine plane, and we will clarify the following property:

the lines in the plane are determined by the points in the plane.

That is, a line is determined by **certain subsets of the set of points, of minimum size 2.**

Definition 2

If a line passes through n points of the set S , it is called **n -rich**.

We restate **Problem 1** as the following theorem, then proceed to prove it.

Theorem 1.1 (Sylvester–Gallai Theorem)

Every such set S determines at least one ordinary line.

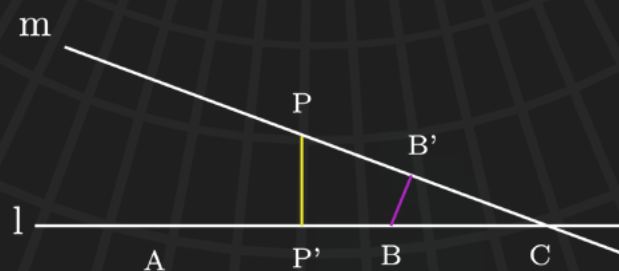
On first instinct, proof by contradiction seems to be the way to go. And it was, for the American mathematician **Leroy Milton Kelly** (1914–2002), although this proof was pretty pervasive, and went undiscovered for decades, it is a really short and simple one.

Let us first look at Kelly’s proof, which is done with elementary geometrical construction, proving the theorem by contradiction.

2 Kelly’s Proof

Proof. Since we have an arrangement of a finite number of points in our plane, the set of lines determined by these points is also finite.

This fact ensures that we can find a point-line pair in the arrangement, such that the perpendicular distance of the point, from the line, is the **minimal perpendicular distance between any point-line pair** in the arrangement. Let P be the point and l be the line in this point-line pair.



Now we draw a perpendicular PP' onto l . There must exist at least one point on either side of P' , on l . We have two cases:

1. l is ordinary.
2. l is not ordinary, it is n -rich for some $n > 2$.

(We know we require at least two points to determine a line, hence 0-rich or 1-rich lines cannot be defined.) We assume Case 2 to be true. Then, there must be more than one point on the left or right of P' . Let us say there are two points B and C to the right of P' , such that B is the first point on the right of P' , and C is some other point.

Now, we draw a line m , through PC . We draw a perpendicular from B onto m . Observe that this perpendicular BB' , will have a shorter length than PP' . Then we have found a point-line pair, namely B and m , such that the perpendicular distance is even lesser than PP' .

But, this contradicts our choice of l and P as the closest point-line pair. Hence, Case-I must be true, and l is proved to be ordinary. $\square$

This is definitely one of the easiest, most elegant proofs I've come across, and is very fun to show others as well. The fact that this simple observation slipped past people for over 4 decades, while much more convoluted proofs like the one by Melchior (1941) were discovered, is pretty amazing to me.

Gallai's Proof to be continued in **DONUT** 6th Edition.



Extending the Factorial Beyond Positive Integers

Shibraj Sahu, IISER Kolkata

Returning to Euler's Infinite Product definition

Recalling the infinite product definition of the factorial, we realise that :

$$\Gamma(x) = \frac{1}{x} \prod_{k=1}^{\infty} \left(\frac{1 + \frac{1}{k}}{1 + \frac{x}{k}} \right)^x$$

Apparently, Euler ingeniously reached at the infinite product formula for the factorial, which is

$$\left[\left(\frac{2}{1} \right)^m \frac{1}{m+1} \right] \cdot \left[\left(\frac{3}{2} \right)^m \frac{2}{m+2} \right] \cdot \left[\left(\frac{4}{3} \right)^m \frac{3}{m+3} \right] \cdots = m!$$

The reader can verify this equation by cancelling out all the common factors which appear in the top and bottom of L.H.S. of the equation. This can also be proved with the help of mathematical induction.

The above equation is more conventionally written as:

$$m! = \lim_{n \rightarrow \infty} \frac{n! \cdot (n+1)^m}{(m+1)(m+2) \cdots (m+n)}$$

For a fixed integer m , as the integer n increases, we have:

$$\lim_{n \rightarrow \infty} \frac{n! \cdot (n+1)^m}{(n+m)!} = 1 \quad (3)$$

If m is not an integer, then it is not possible to say whether this equation (3) is true because we have not yet (in this section) defined the factorial function for non-integers. However, we do get a unique extension of the factorial function to the non-integers by insisting that this equation (3) continue to hold, when the arbitrary integer m is replaced by an arbitrary real number x (except for negative integers),

$$\lim_{n \rightarrow \infty} \frac{n! \cdot (n+1)^x}{(n+x)!} = 1$$

Multiplying both sides by $(x-1)!$ gives

$$\begin{aligned} \frac{1}{x} \lim_{n \rightarrow \infty} \frac{n!}{(n+x)!} \cdot \frac{x!}{(n+1)^x} (n+1)^x &= (x-1)! \\ &= \Gamma(x) \end{aligned}$$

$$\implies \Gamma(x) = \frac{1}{x} \lim_{n \rightarrow \infty} \frac{n! \cdot (n+1)^x}{(x+1)(x+2) \cdots (x+n)} \quad (4)$$

Now, let us try to manipulate this equation further. We see that,

$$(n+1)^x = \left[\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \frac{n+1}{n} \right]^x = \prod_{k=1}^n \left(1 + \frac{1}{k} \right)^x \quad (5)$$

$$(x+1)(x+2) \cdots (x+n) = 1 \cdot \left(1 + \frac{x}{1} \right) \cdot 2 \cdot \left(1 + \frac{x}{2} \right) \cdots n \cdot \left(1 + \frac{x}{n} \right) = n! \prod_{k=1}^n \left(1 + \frac{x}{k} \right) \quad (6)$$

From (4), (5), and (6), we get:

$$\begin{aligned} \Gamma(x) &= \lim_{n \rightarrow \infty} \frac{1}{x} \prod_{k=1}^n \frac{\left(1 + \frac{1}{k} \right)^x}{1 + \frac{x}{k}} \\ \implies \Gamma(x) &= \frac{1}{x} \prod_{k=1}^{\infty} \frac{\left(1 + \frac{1}{k} \right)^x}{1 + \frac{x}{k}} \end{aligned}$$

with this, we have finally verified the Euler's infinite product definition of gamma function.

Final Destination!

In this long journey till now, we have seen the gamma function

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$

many times. And I am sure, you must have wondered – "how did we get this beautiful integral to represent the Gamma function?". It's really not so easy to peek inside the brain of the great Euler and find the origin of this integral.

Nonetheless, in this section, we will try to find an intuitive idea behind this beautiful integral and finally end our journey.

We know,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

As we take n^{th} order derivative of x^n , we can see something interesting.

$$\frac{d^n}{dx^n}(x^n) = n(n-1)(n-2)\cdots 2\cdot 1 = n!$$

Now, when we take n^{th} order derivative of $-x^{-1}$:

$$\frac{d^n}{dx^n}(-x^{-1}) = \frac{(-1)^{n+1}}{x^{n+1}}n!$$

We will try to write $-x^{-1}$ in terms of exponential function as we know that

$$\frac{d}{dx}(e^{tx}) = te^{tx}$$

which is beneficial for us to get our desired result.

We know,

$$\int e^{xt} dt = \frac{e^{xt}}{x} + c$$

If the limit for the above integral is from $0 \rightarrow \infty$, we get

$$\int_0^\infty e^{xt} dt = \lim_{r \rightarrow \infty} \frac{e^{rx}}{x} - \frac{1}{x} = -\frac{1}{x}, \quad x < 0$$

$$\therefore \frac{d^n}{dx^n}(-x^{-1}) = \frac{d^n}{dx^n} \int_0^\infty e^{xt} dt = \frac{n!}{(-x)^{n+1}}, \quad x < 0$$

$$\implies \int_0^\infty t^n e^{xt} dt = \frac{n!}{(-x)^{n+1}}$$

Now, when $x = -1$

$$\int_0^\infty t^n e^{-t} dt = n!$$

$$= \Gamma(n+1)$$

I hope, finally you can say that you have got some intuition behind the origin of the integral definition of the Gamma function. Note that, we showed it only for positive integers, but this idea can be extended to real numbers as well as complex numbers. With this, we have come to the end of our journey of unravelling the wonderful factorial function (or I should say the Gamma function). We have come to realise that Gamma function is the generalized function behind the factorial, which is well defined for all complex numbers except the negative integers.

The article started with the motivation to find the extension of factorial from the discrete world of natural numbers to the continuous world of real numbers and thus, justify the graph of $x!$. But, in the end, we have found an amazing Gamma function which has loads of applications in many areas of mathematics and physics.

References

- Davis, P. J. (1959). “Leonhard Euler’s Integral: A Historical Profile of the Gamma Function”. *American Mathematical Monthly*.
- Beals, Richard; Wong, Roderick (2010). *Special Functions: A Graduate Text*. Cambridge University Press. p. 28. ISBN 978-1-139-49043-6. Extract of page 28.
- Bonvini, Marco (9 October 2010). “The Gamma function” (PDF).
- Unravelling the mystery of this math miracle – BriTheMathGuy
- Askey, R.; Roy, R. (2010), “Gamma function”, in Olver, Frank W. J.; Lozier, Daniel M.; Boisvert, Ronald F.; Clark, Charles W. (eds.), *NIST Handbook of Mathematical Functions*, Cambridge University Press,

When Simple Machines Go Wildly Out of Control

Christopher, BS-MS Batch'23

Introduction

In 1962, the mathematician Tibor Radó introduced a simple yet profound question: How much can a simple computer program (modeled by a Turing machine) achieve before reaching a halting state? This question, now known as the Busy Beaver Problem, led to one of the most striking examples of non-computability. This problem has revealed some fundamental limitations on what algorithms can determine.

The Setup: Turing Machines

Imagine a machine with:

- an infinite strip of tape filled with symbols (just 0s and 1s) extending in both directions
- a head that reads and writes new symbols over the current symbol.
- and a set of rules that tell it what to do depending on the current symbol and state.

These are called Turing machines, and they are among the simplest models of computers. Despite their simplicity, they can still do anything any modern computer can do, if you give them enough time.

All Turing machines either get stuck in infinite loops and run forever, or they perform a number of rewrites, and then eventually halt. That is, they reach a special “halting” state and stop. The Busy Beaver problem is about such machines: ones that do halt.

The Big Question

Suppose we fix the number of rules to be n (i.e, the machine has n states) and we start them on a blank tape (a tape filled completely with 0 s).

Now ask:

"Among all the machines with n states that eventually halt, which one goes over the most number of steps before halting?"

This machine is called the Busy Beaver for that value of n (denoted by $BB(n)$). The number of 1 s it writes is denoted by $\Sigma(n)$, and the number of steps it takes before halting is denoted by $S(n)$.

$S(n)$ and $\Sigma(n)$ grow quickly with n . But how quickly?

The Explosion

Here are all the known Busy Beaver values so far:

n	$\Sigma(n)$	$S(n)$
1	1	1
2	4	6
3	6	21
4	13	107
5	4098	47176870.

By the time we hit just 5 states, the numbers explode. Past $n = 5$, we do not even know the exact values anymore. For 6-state machines, one known candidate runs for over 7.4×10^{36534} steps, a number vastly larger than the number of particles in the universe ($\sim 10^{80}$). This is not just big, it is beyond anything we can comprehend.

However, this is not just because we have not found the answer yet. The twist is that there is no algorithm that can compute $\Sigma(n)$ for all n .

Why Not?

Here is the intuition:

Alan Turing proved in 1936 that there is no universal way to tell whether a given machine will halt (this is known as Turing's Halting Problem). The proof goes like this:

Suppose there exists an algorithm, which we call Mufasa, that takes any program P and correctly predicts whether P halts on that input.

Now, construct a new program, let's call it Scar, which is defined as follows:

- Given an input program P , Scar uses Mufasa to ask: "Does P halt when run?"
- If Mufasa says "Yes," then Scar loops forever (does not halt).
- If Mufasa says "No," then Scar halts immediately.

Finally, consider what happens when we input $P = \text{Scar}$ into Mufasa:

- If Mufasa predicts that Scar halts, then Scar loops forever, resulting in a contradiction.

- If Mufasa predicts that Scar does not halt, then Scar halts immediately, again contradicting the prediction.

This shows that Mufasa cannot exist. Therefore, there is no algorithm that can decide, for every program and input, whether the program halts.

Since finding the Busy Beaver requires knowing exactly which machines halt, by Turing's proof, it turns out that the Busy Beaver problem for a general n is undecidable.

In fact, the problem gives us an example of a function that grows faster than any computable function you can define—faster than exponentiation, faster than the Ackermann function, faster than anything your computer can be programmed to compute.

That is what makes it so intriguing. It is a simple problem to state, but it pushes past the limits of computability.

A Digital Wild West

Despite this wall, people have tried to push the known values as far as possible. The search for such machines has become an ongoing effort among hobbyists, researchers, and online communities. Some people build custom simulators. Others use tricks from logic to prove whether a machine will ever halt. It is part puzzle hunt, part scientific expedition.

In 2022, amateur Busy Beaver researcher Tristan Stérin set up <https://bbchallenge.org/> with the goal of finding $BB(5)$. Finally in 2024, with the help of multiple contributions from various users across the world, $BB(5)$ was finally proven to be the machine that writes 4,098 ones and takes over 47 million steps to halt.

Why It Matters

At first glance, the Busy Beaver problem might seem like an obscure puzzle about strange little machines. But it touches some of the deepest ideas in computer science and mathematics.

For one, it gives a concrete window into non-computability. While Turing's Halting Problem is abstract, Busy Beavers are explicit machines whose behavior we still cannot predict. Even small ones can run for unimaginably long periods, showing that their unpredictability is by virtue of the computation itself.

But the Busy Beaver function $S(n)$ is not just about computability. It touches the foundations of mathematics. Any conjecture that can be disproven by a single counterexample (called a Π_1^0 statement, like Goldbach's conjecture) can, in theory, be resolved by simulating an n -state Turing machine that searches for such a counterexample. If the machine halts, the conjecture is false; if it runs forever, the conjecture holds. Knowing the exact value of $S(n)$ would allow us to verify this. However, this may not be practical for two reasons.

1. It is extremely hard to find busy beavers past $n = 5$, due to the sheer amount of Turing machines.
2. Since $S(n)$ gets very large very quickly, computing this may not be feasible.

Busy Beavers also have ties to other integral parts of math. For example, no computably axiomatized, arithmetically sound theory can prove all values of $S(n)$. For each such theory T , there is a threshold n_T beyond which $S(n)$ cannot be proven within T , due to Gödel's second incompleteness theorem. This creates a hierarchy of theories based on their "proving power". For example, Turing machines have been constructed

that halt if and only if major conjectures like ZFC consistency, the Riemann Hypothesis, or Goldbach's conjecture are false, linking Busy Beaver values directly to many deep open problems in mathematics. So while no one builds software with Busy Beavers, they are still essential in understanding the edge of what computation, and even mathematics, can ultimately know.

Final Thoughts

The Busy Beaver problem shows us how complicated behavior can emerge from simple rules. Even when the machine only has a few states and follows a tiny set of instructions, it can behave in ways we cannot predict, and sometimes cannot even understand. It is a reminder that mathematics is beyond mere computation.



Emergent Phenomena from Randomness

Aditya panicker, BS-MS Batch'22

Introduction

You can model a surprisingly large number of natural processes as “random” or “probabilistic.” But here's the kicker: when you gather a bunch - or an ensemble - of things all obeying these probabilistic rules, the collective behavior often turns out to be anything but random. In fact, something pretty remarkable happens - **order emerges from chaos**. It's like nature's own Yin-Yang paradox, where deterministic patterns pop out of fundamentally random ingredients.

But before we get too deep into philosophical rabbit holes, let's cut to the chase with an example:

Exhibit A: The Galton Board and the Central Limit Theorem

Ah yes, the trusty Galton Board - the physicist's favorite party trick for showing how a Gaussian distribution (a.k.a. the bell curve) magically appears from a bunch of coin-flip-style decisions. It's a hands-on, pin-and-ball version of Pascal's Triangle, which, if you've never seen, might suggest you're stuck in some weird parallel dimension (let's be honest, “living under a rock” is kind of outdated).

Here's how it works: balls are dropped from the top of the board, bouncing off pegs as they fall. Each bounce is like a coin flip - left or right. For a single ball, its final position at the bottom is unpredictable. But - and here's the cool part - drop a lot of balls (say, 1000), and you'll start seeing a pattern. The balls tend to pile up near the center. Why? Because there are way more paths that lead to the middle than to the extremes. The ends are kind of lonely - not many ways to get there.

Stack up all these outcomes and overlay them - boom! - a Gaussian distribution of Random Variables (which, by the way, are neither particularly “random” nor “variable” - they're functions, but we like to give them dramatic names) appears. Randomness giving birth to order. And this isn't just a neat trick - it's a real, powerful result in probability theory known as the **Central Limit Theorem** (or in special cases, the **Laplace–De Moivre Theorem**).

The math behind this starts with a binomial distribution, where each path corresponds to a sequence of left-right decisions:

$$P(k; n) = \binom{n}{k} p^k (1-p)^{n-k}$$

As $n \rightarrow \infty$, and assuming $p = 0.5$, the binomial distribution smooths out into a Gaussian (normal) distribution:

$$\lim_{n \rightarrow \infty} P(k; n) \approx \frac{1}{\sqrt{2\pi np(1-p)}} \exp\left(-\frac{(k - np)^2}{2np(1-p)}\right)$$

Welcome to the Central Limit Theorem (CLT) - the ultimate statistical shape-shifter. It tells us that sums (or averages) of many independent random variables tend toward a normal distribution, regardless of the original distribution. It's the mathematical engine behind why "bell curves" show up literally everywhere.

Exhibit B: Thermodynamic Properties

Now here's where things get spicy. That Galton Board analogy? It's more than a math toy - it taps into a deeper truth about nature. A single molecule doesn't have properties like temperature or pressure. It's just doing its own thing, buzzing around. But round up a gang of 10^{23} (hello, Avogadro!) of them, and suddenly - bam! - thermodynamic properties start to emerge.

This is the central idea of **statistical mechanics**. Microscopic randomness, when viewed on a massive scale, produces reliable, stable macroscopic behavior. The molecules might be going berserk individually, but together, they behave in a way that's predictable and even elegant - this is where the magic happens - out of a chaotic soup of bouncing particles, well-defined macroscopic properties rise like a phoenix. It's like statistical sorcery.

In short, the whole really is greater than the sum of its parts. And it's not just poetic - it's physics.

Bonus Concept: Entropy and Emergence

Entropy, the legendary bad-boy of thermodynamics, measures disorder - or more precisely, the number of microstates corresponding to a macrostate:

$$S = k_B \ln \Omega$$

Where Ω is the number of accessible microstates.

Here's the twist: more randomness at the micro level often means more predictability at the macro level. That's how statistical mechanics flips the script - disorder leads to order, when viewed from the right perspective.

Closing Thoughts

Emergent phenomena remind us that randomness is not the enemy of order - it's often *the source* of it. From falling balls to bouncing molecules to flipping spins, systems made of chaos can display breathtaking simplicity and structure when scaled up.

So the next time someone says, "It's all random," you can grin and reply:

"Yeah, and that's exactly why it all makes sense."



Cracking the Code: How Primes Make Cryptography Unbreakable

Damini Das, BS-MS Batch'22

Imagine working at a bank responsible for securing online transactions. Your task is to implement a cryptographic protocol to protect sensitive information between the bank's servers and its customers' devices. This is where prime numbers come to the rescue.

A number that is the product of exactly two (not necessarily distinct) prime numbers is called a **semi-prime**. While generating a semi-prime is computationally simple, factoring it into its prime components is much more difficult, especially when the primes are large. This hardness is a cornerstone of the RSA encryption algorithm.

The RSA algorithm uses two large prime numbers, p and q , to generate a public key and a private key. The public key can be shared openly, while the private key must remain confidential. Because p and q are large, factoring their product $N = pq$ is computationally infeasible for attackers.

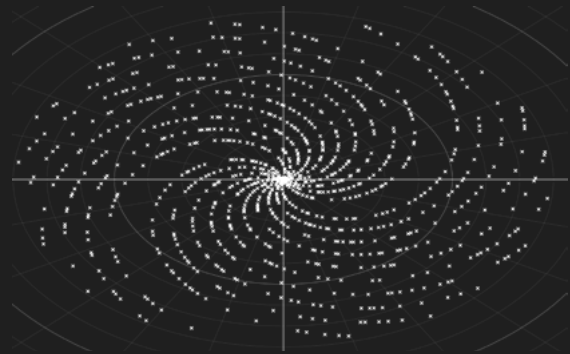
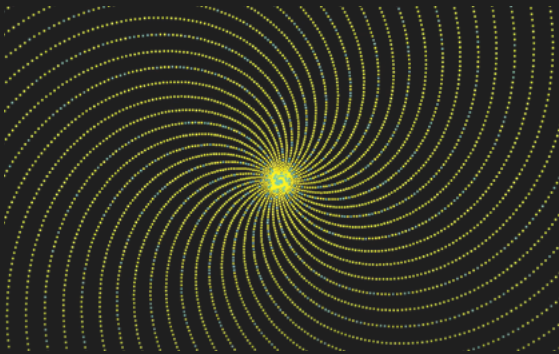
Cryptographic systems heavily rely on modular arithmetic (also known as clock arithmetic), which wraps integers around a fixed modulus n . In this system, we work with remainders:

$$a \equiv b \pmod{n} \iff n \mid (a - b)$$

A familiar example is a 12-hour clock, where $14 \equiv 2 \pmod{12}$.

Prime numbers exhibit intriguing patterns in modular arithmetic. When plotted in polar coordinates $(r, \theta) = (r, r)$, with $r \in \mathbb{N}$, natural numbers form spiral arms. If we count by increments of 44, the number of arms in this spiral is 44, corresponding to residue classes modulo 44:

$$\text{Spiral arms: } 44k, 44k + 1, \dots, 44k + 43$$



However, not all arms contain primes. For example:

- Even-numbered arms like $44k + 2, 44k + 4, \dots$ are discarded because they are not relatively prime with respect to 44.
- Multiples of 11 such as $44k + 11$ and $44k + 33$ are excluded as well.

Thus, 22 even arms + 2 arms divisible by 11 = 24 arms are excluded, leaving 20 valid arms that may contain primes.

The **Euler's totient function** $\phi(n)$ counts the number of positive integers less than n that are relatively prime to n . That is:

$$\phi(n) = |\{k \in \mathbb{Z}_{>0} \mid 1 \leq k < n, \gcd(k, n) = 1\}|$$

The numbers that are relatively prime to n are called **totatives** of n .

Proof Sketch. Let:

$$S = \{x_1, x_2, \dots, x_{\phi(n)}\} \text{ be the set of totatives of } n$$

Then:

$$B = \{ax_1 \bmod n, ax_2 \bmod n, \dots, ax_{\phi(n)} \bmod n\}$$

Since a is coprime to n , each $ax_i \bmod n$ is also a totative, and the set B is a permutation of S . Therefore,

$$\prod_{i=1}^{\phi(n)} ax_i = a^{\phi(n)} \prod_{i=1}^{\phi(n)} x_i \equiv \prod_{i=1}^{\phi(n)} x_i \bmod n$$

Cancelling $\prod_{i=1}^{\phi(n)} x_i$, we get:

$$a^{\phi(n)} \equiv 1 \bmod n$$

Another widely used method for key exchange over insecure channels is the **Diffie-Hellman protocol**:

- **Public Parameters:**
 - A large prime p
 - A base (generator) $g \in \mathbb{Z}_p^*$
- **User A (you):**
 - Chooses a private key $a < p$
 - Computes $A = g^a \bmod p$, and sends A to User B
- **User B (friend):**
 - Chooses a private key $b < p$
 - Computes $B = g^b \bmod p$, and sends B to User A
- **Key Derivation:**
 - User A computes $s = B^a \bmod p$
 - User B computes $s = A^b \bmod p$

Since:

$$B^a \equiv (g^b)^a \equiv g^{ab} \pmod{p} \quad \text{and} \quad A^b \equiv (g^a)^b \equiv g^{ab} \pmod{p}$$

Both users arrive at the same shared secret key $s \equiv g^{ab} \pmod{p}$. □

The fascinating interplay between prime numbers and modular arithmetic forms the mathematical backbone of modern cryptography. From spiral plots to encryption keys, primes enable secure communication in our digital world. So, the next time you browse the web or send a message, remember—the invisible guardians securing your data are simple, elegant prime numbers.

π-WEEK CELEBRATIONS

At IISER Thiruvananthapuram, π-Week is an annual celebration of the π-day, March 14th, highlighting the elegance and enjoyment found in mathematics. The week featured a diverse lineup of events, including contests, talks and screenings, all designed to foster a vibrant mathematical spirit and encourage an appreciation for the subject.

π-Bytes

March 14, 2025

As part of this year's π-Week celebrations, participants engaged in a π remembering contest that put their memory and focus to the test. Each contestant attempted to write down as many digits of Pi as they could, one at a time, with only a brief pause between each digit. The event saw participants pushing their limits, until a single mistake brought their turn to



Image Credit: Mukesh Mithun B23

an end. With exciting prizes awaiting the winner, the competition was both intense and fun.

Winner: Pearl (B23)
Runner up: Hari Govind K J (B22)

Freehand Bourbaki

March 14, 2025

The competition challenged the participants to draw the most perfect freehand circle in a single stroke. With no tools allowed and no second chances, the competition tested steady hands and sharp focus. Each circle was judged on its roundness, smoothness, and precision. The event stood out as a unique and engaging highlight, with exciting prizes awaiting the most perfect circle.

Winner: Amal Krishna Sajeesh (B23)
Runner up: Padmanabhan Kannan (B21)

Flash Talk

March 15, 2025

This year's π -Week was commemorated by "Flash-Talk" where students were invited to give a brief talk on any topic related to math history. Bharathkrishna S spoke about 'Symbolica: A brief history on mathematical symbols and notation', while Christopher Rao gave a talk on 'Women in Math: A Short History'. Each talk shed light into the different aspects of the history of mathematics, celebrating both its depth and diversity.



π -uiz 6.0

March 16, 2025

As part of π -Week 2025, CMIT, in collaboration with QSI, hosted the sixth edition of its flagship quiz competition, π -uiz 6.0. Packed with fun and brain-wracking mathematical questions, the event saw enthusiastic participation from teams of up to two members. The quiz featured two rounds - prelims and finals. The top six teams from the prelims advanced into the finals, where they continued their battle to be the winners of π -UIZ 6.0. The quiz was a lively and memorable celebration of mathematical wit and the spirit of friendly competition.



Image Credit: Mukesh Mithun B23

Winner: S V Roshini (B22), Bharath Krishna (iPhD 21)
Runner up: Mukesh Mithun A D (B23), Ashish Kujur (PhD 23)

Movie Screening

March 18, 2025

One of the highlights of this year's festivities was a special screening of 'The Man Who Knew Infinity'. The film, based on the life of the brilliant Indian mathematician Srinivasa Ramanujan, captivated the audience with its portrayal of his extraordinary contributions to mathematics and his journey to the prestigious University of Cambridge. It was a fitting tribute to π -Day, blending cinematic storytelling with the spirit of mathematical exploration.



E- π -phany 4.O

March 29, 2025

The math-themed treasure hunt was held on Discord, challenging participants with puzzles that demanded sharp problem-solving skills and seamless teamwork. The contest saw active participation of teams of atmost three, being a fun and engaging way to celebrate π -Day.

Winner: Aryan Bhatia (B21)
Runner up: Pranav Manoj (B23)

TALKS AND DISCUSSIONS

Tangent Tuesdays

Aparna Bade

Dinitz problem

13 August 2024, 9 pm.

Tanisha Talekar

Quiver me timbers

13 August 2024, 9:30 pm.

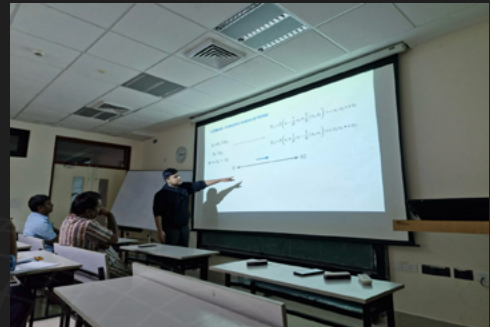


Image Credit: Mukesh Mithun B23

Suhana Nujum



Leveraging Single-Cell RNA Sequencing and Machine Learning to Explore MicroRNA Regulation in Kidney Transplant Rejection

20 August 2024, 9 pm.

Sinta Maria Siby

Optimising tissue network configurations

20 August 2024, 9:30 pm.

Yogesh Kumar KC

Large but finite N

27 August 2024, 9 pm.

Soumyadip Niyogi



Analysis of dynamical systems in Cosmology

27 August 2024, 9:30 pm.

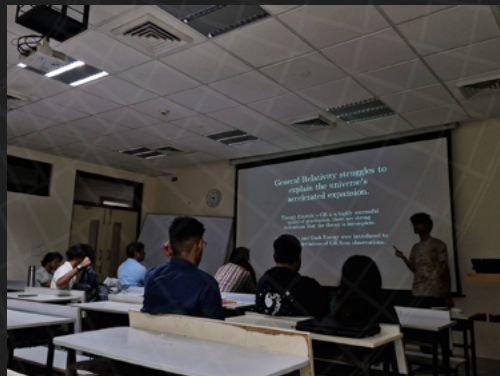


Image Credit: Mukesh Mithun B23

Visesh Jyothi A

Conformal Maps and the Riemann Mapping Theorem: Bridging Complex Domains

29 August 2024, 4 pm.

Arjun V Nair

Some secrets of the Helmholtz equation

Part 1 - 29 August 2024, 4:30 pm.

Part 2 - 3 September 2024, 4 pm.

Subhradeep Ghosh

p -adic Fields

10 September 2024, 9 pm.

Talk Series

Ashish Kujur

To infinity and beyond with ordinals

4 February 2025, 9 pm.

Bharath Krishna S

The many dynamics on(f) the shift space

6 February 2025, 9 pm.

Tanisha Mandar Talekar

Quiver representations

11 March 2025, 9 pm.



*One cat doodle
because why not!*

Subhradeep Ghosh

Bratteli Diagrams

18 April 2025, 9 pm.

Peer Discussions

Mukesh Mithun A D

Riemann's theorem on conditionally convergent series

10 April 2024, 6 pm.

Completeness of finite and infinite dimensional Normed Linear Spaces

29 October 2024, 9 pm.

Alumni Talks

Kalin Krishna (Doctoral student at the University of Gottingen, Germany)
Alumni, BS-MS, Batch '16 IISER TVM

Informal Q&A session with Alumni

26 September 2024.

Faculty Talks

Dr. Vaibhav Vaish
IISER Mohali

On the Pappus' theorem in geometry

27 November 2024.

Dr. Manjil Saikia
Ahmedabad University

Cataland: A Romance of Many Bijections

27 November 2024.

Dr. Shailesh Shirali

Vignettes in Number Theory, Analysis and Ancient Indian Mathematics

28 November 2024.

Other Activities

Career Guidance

22 August 2024.

This event is an informal discussion designed to help Mathematics majors explore opportunities for internships and PhD programs.

Review series

25 November 2024.

The review series was conducted to help the students to go through their syllabus in depth and beyond. It also provided a platform for the doubt clearance of students.

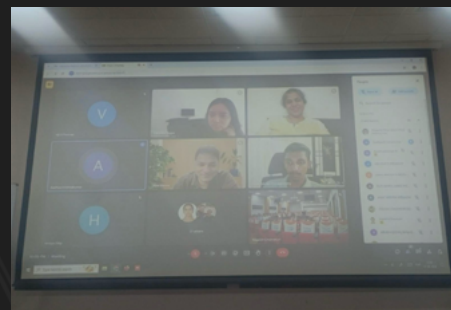


Image Credit: Mukesh Mithun B23

FRONTIER SYMPOSIUM '25

From January 17th to 19th, 2025, the School of Mathematics of IISER Thiruvananthapuram hosted the annual Mathematics Frontier Symposium at the LHC Jupiter, with the support of Club of Mathematics IISER Thiruvananthapuram (CMIT).

This event aimed to introduce students to a wide range of mathematical fields through expert lectures. The symposium kicked off with an inauguration ceremony led by Prof. J N Moorthy, Director of IISER Thiruvananthapuram, and featured insightful talks by renowned mathematicians from within and outside the institute. These lectures provided a comprehensive overview of various mathematical disciplines, inspiring further exploration. The event also included dedicated Q&A sessions, creating a dynamic learning environment where students could engage with speakers, deepen their understanding, and spark curiosity in different mathematical areas.



Image Credit: Mukesh Mithun B23



PEOPLE WHO FRIED THIS DONUT

Meet The Team

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All *paws* on deck!